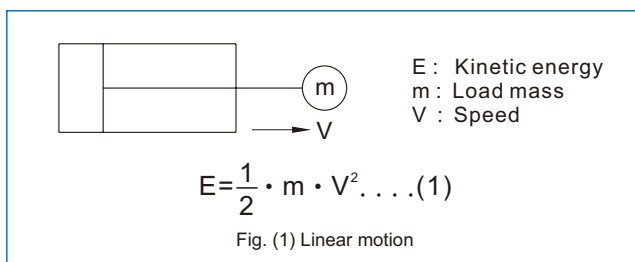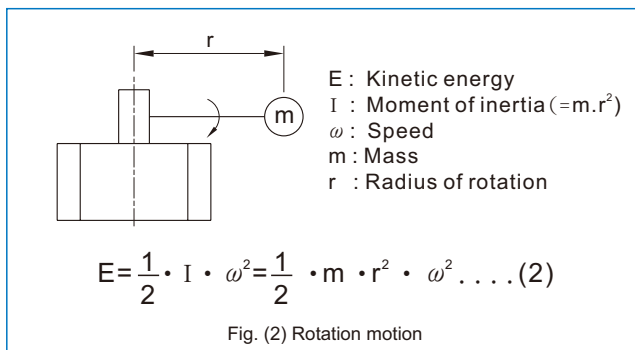# Moment of inertia
ROTARY ACTUATOR

- The load will create inertial forces (kinetic energy) when moving the load with Rotary Actuator. In order to stop the moving load, It is necessary to use stopper or Shock Absorbers to absorb the kinetic energy of load.
- The moving load with actuator can be distinguished as following
  1. Linear motion (air cylinder), Fig.(1)
  2. Rotation motion (rotary actuator), Fig.(2)
- Calculate the kinetic energy by using the formula in FIG.

Linear motion

E : Kinetic energy
m : Load mass
V : Speed

$$E = \frac{1}{2} \cdot m \cdot V^2 \ldots (1)$$

Fig. (1) Linear motion

Rotation motion

E : Kinetic energy
I : Moment of inertia $(=m.r^2)$
ω : Speed
m : Mass
r : Radius of rotation

$$E = \frac{1}{2} \cdot I \cdot \omega^2 = \frac{1}{2} \cdot m \cdot r^2 \cdot \omega^2 \ldots (2)$$

Fig. (2) Rotation motion

- For linear motion, if the velocity V of formula (1) is constant, the kinetic energy E and mass m is proportional ; The rotation motion, formula (2) shows even the angular velocity ω and mass m is constant, kinetic energy E will also proportional with r². Therefore, even the mass is small but the rotation radius r is large, when the moment of Inertia I=m·r² is large, kinetic energy E will becomes larger, it will cause bearing damage or other accidents.
- Therefore when there is a rotation motion, the product selection should be base on moment of Inertia instead of mass.

Moment of inertia

- Moment of inertia shows, it is not easy to rotate the stationary object; the same which means it is difficult to stop the rotating object.
- Rotary Actuators in the allowable kinetic energy has its limitations, it can be calculated moment of Inertia to calculate minimum rotation time limit. The calculation of moment of Inertia described as following:

$$I = m \cdot r^2$$

m : Mass
r : Radius of rotation
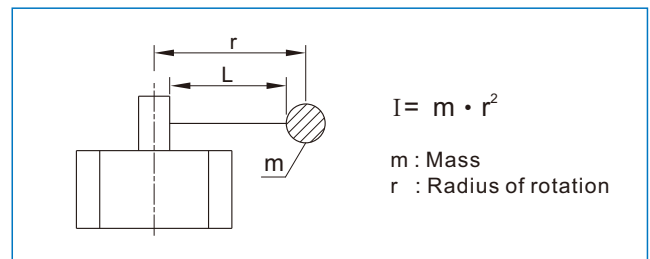
Above figure represents the moment of inertia for the distance r from rotary shaft to mass m of the object.
The formula for moment of Inertia is not the same if the shapes of the object are different.
The following examples are the calculated on the basis of specific moment of inertia.
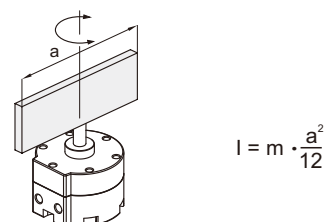
1.Thin shaft

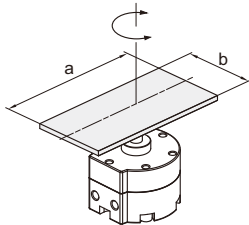Position of rotational axis: Perpendicular to the shaft through the center of gravity.

$$I = m \cdot \frac{a^2}{12}$$
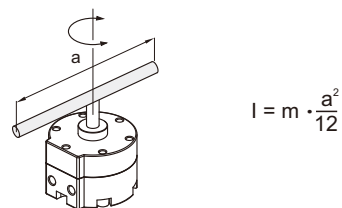
2.Thin rectangular plate

Position of rotational axis: Parallel to side b and through the center of gravity.

$$I = m \cdot \frac{a^2}{12}$$

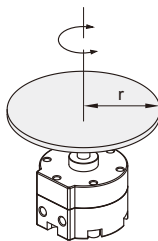### 3. Thin rectangular plate (Including rectangular parallelepiped)

Position of rotational axis: Perpendicular to the plate through the center of gravity.

$$I = m \cdot \frac{a^2 + b^2}{12}$$

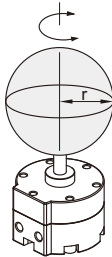### 4. Round plate (Including column)

Position of rotational axis: Through the center axis.

$$I = m \cdot \frac{r^2}{2}$$
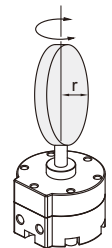
5. Solid sphere

Position of rotational axis: Through the center of diameter.
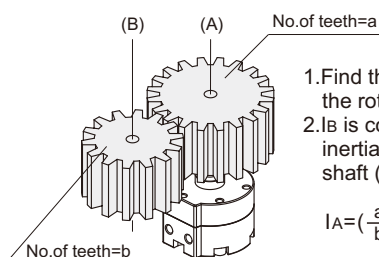

$$I = m \cdot \frac{2r^2}{2}$$

6. Thin round plate

Position of rotational axis: Through the center of diameter.

$$I = m \cdot \frac{r^2}{4}$$

7. Gear transmission

(B)   (A)   No.of teeth=a

No.of teeth=b

1. Find the moment of inertia I_B for the rotation of shaft (B).
2. I_B is converted to the moment of inertia I_A for the rotation of the shaft (A).

$$I_A = \left(\frac{a}{b}\right)^2 \cdot I_B$$

Use the following formula to calculate the rotation time

$$t \geqq \sqrt{\frac{2 \cdot I \cdot \theta^2}{E}}$$

- t : Rotation time (s)
- E : Kinetic energy (J)
- I : Moment of inertia (kg.m²)
- θ : Rotation angle (rad)

The meaning of this formula is the critical rotation time for not cause damage of the cylinder. Therefore the rotation time must be set on or over the t seconds calculated in above formula.

After calculated the moment of inertia by load shape, use the following formula to calculate the kinetic energy of the load.

$$E = 1/2 \cdot I \cdot \omega^2$$

- E : Kinetic energy (J)
- I : Moment of inertia (kg.m²)
- ω : Angle speed (rad/s)

Angle speed

$$\omega = 2\theta / t \ldots (1)$$
$$\omega = \theta / t \ldots (2)$$

- t : Rotation time (s)
- I : Moment of inertia (kg.m²)
- θ : Rotation angle (rad)

However, when the rotation time for 90° becomes longer than 2 seconds, use formula (2).

Calculation example

Load form: Cuboid

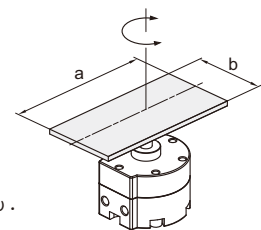|	
Rotation angle θ : 180°	Length of a part : 0.12 m
Rotation time t : 1 s/180°	Length of b part : 0.06 m
	Mass (m) : 0.1 kg

$$I = m \cdot \frac{a^2 + b^2}{12}$$

(Step 1) Find the angle speed ω.

$$\omega = \frac{2\theta}{t} = \frac{2}{1} \times \pi = 6.28 \text{ rad/s}$$

(Step 2) Find the moment of inertia I.

$$I = m \cdot \frac{a^2 + b^2}{12}$$
$$= 0.1 \times \frac{144 \times 10^{-4} + 36 \times 10^{-4}}{12}$$
$$= 1.5 \times 10^{-4} \text{ kg.m}^2$$

(Step 3) Find the kinetic energy E.

$$E = \frac{1}{2} \cdot I \cdot \omega^2 = \frac{1}{2} \times 1.5 \times 10^{-4} \times 6.28^2$$
$$= 0.002958 \text{ J}$$